A Homogenization-based Phase Field Approach to Fracture

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Abstract

The phase field method [2] has gained popularity due to its ability to predict crack nucleation, propagation, and branching without extra criteria. This approach works by minimizing a total energy functional with the displacement field and phase field (0=intact material, 1=crack) as arguments, and eliminates the cumbersome geometric tracking.
However, each of the prevailing models [1,5] predicts a different crack path even under certain simple loadings. We apply the homogenization theory to construct a phase field model, which predicts reasonable crack paths for the three-point bending test and through-crack shear test. We compare the prediction of our model with similar ones [6-8].

Introduction

The prediction of failure mechanisms due to crack initiation and propagation in solids is of great significance for engineering applications. Phase field approached to brittle fracture is based on the variational energy formulation proposed by Francfort [1], which takes the Griffith's theory into account. The phase field modeling of brittle fracture have shown its advantages on simulating complex fracture processes including initiation, propagation, branching and merging of cracks [1,2,4].

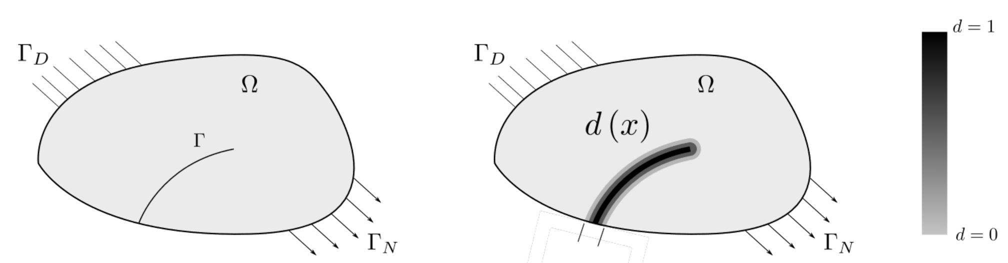


Figure 1. Comparison of a sharp crack and its phase field representation.

Within the models related to Griffith's theory, we focus on these featuring a tension-compression split. That is because cracking under compressive load will lead to unphysical crack propagation patterns without such kind of split. We define a **tension-compression discriminant** to complete such split. Models incorporating a tension-compression split have been proposed by Amor [1], Bourdin[2] and Miehe [4].

Homogenization & Phase Field Model

We model a possibly curvilinear crack as a collection of fictitious small straight cracks with a particular spatial distribution. The behavior of the macroscopic crack is then described by the collection of such small cracks in the RVE, see Figure 2. The proposed phase field is: $\psi[\boldsymbol{\varepsilon}(\boldsymbol{u}), d] = \boldsymbol{H}[\beta(\boldsymbol{\varepsilon})]T + (1 - \boldsymbol{H}[\beta(\boldsymbol{\varepsilon})])C$, where H is the Heaviside function, and $\beta(\boldsymbol{\varepsilon}) = (1-\nu)\varepsilon_y + \nu\varepsilon_x$ switches between the cases in the RVE is in tension or compression, under the given strains $\boldsymbol{\varepsilon}$, and hence $\beta(\boldsymbol{\varepsilon})$ is called **the tension-compression discriminant**.

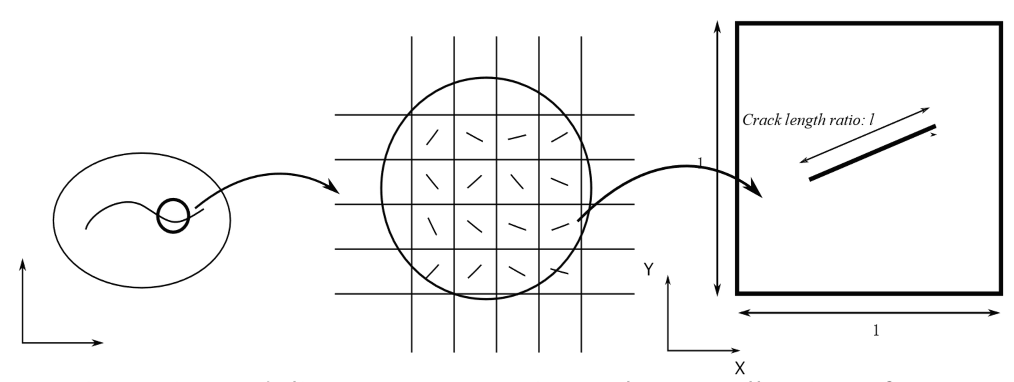


Figure 2. Modeling a macroscopic crack as a collection of fictious small straight cracks, each in an RVE.

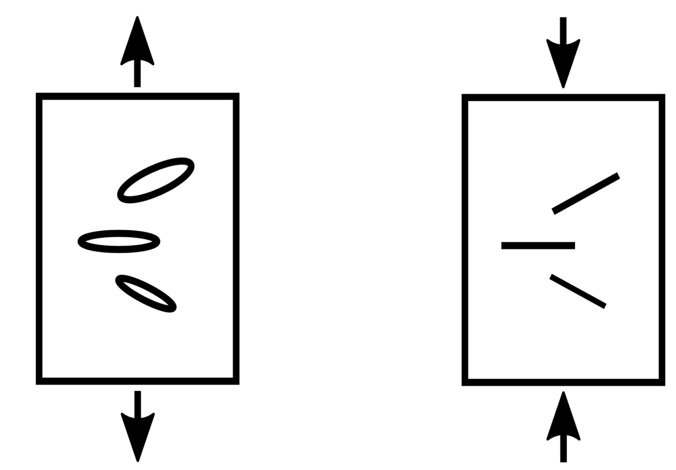


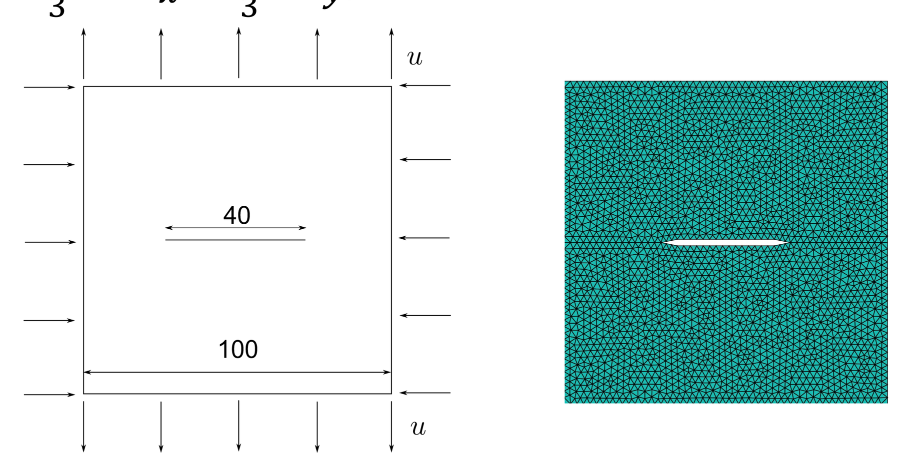
Figure 3. Comparison material behavior under tension and compression.

Benchmark Tests

In uniaxial tension test, our model gives a result similar to Amor's model [1]. In uniaxial compression test, our model gives a result similar to Miehe's model [5]. In shear test, the crack patterns in our model and Miehe's model [5] are more similar and have a much larger angle. In three-point bending test, the stiffness of our model is a little bit stiffer than from Miehe's model [5], only our model and Miehe's model give acceptable results. In through-crack shear test, only our model and Amor's model [1] give reasonable results. Miehe's model shows a stiff response which should not appear for a fully broken plate since it is assumed that there is no friction between the surface of crack.

Poison Effect: Biaxial Loading 1

We add displacement boundary conditions symmetrically: $\boldsymbol{u}_D = -\frac{7}{3}\Delta u\boldsymbol{e}_x + \frac{1}{3}\Delta\boldsymbol{e}_y$, where $\Delta u = 0.01, 0.02$mm.



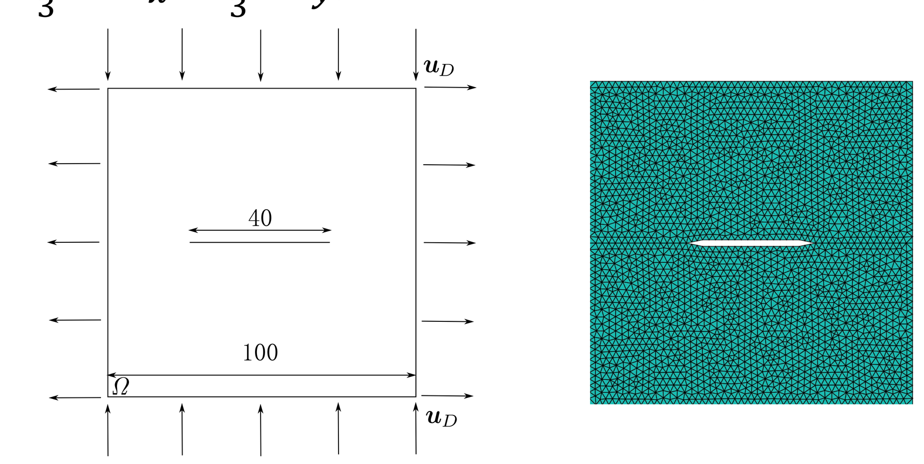
(a) Configuration (b) mesh

Figure 4. Configuration and mesh of the decomposition comparation test 1.

Our model can be simplified as $\sigma_y = \lambda(\varepsilon_x + \varepsilon_y) + 2\mu\varepsilon_y$ in this case, which means it is categorized as **compression**.

Poison Effect: Biaxial Loading 2

This time we add displacement boundary conditions: $\boldsymbol{u}_D = \frac{7}{3}\Delta u\boldsymbol{e}_x - \frac{1}{3}\Delta\boldsymbol{e}_y$, where $\Delta u = 0.01, 0.02$mm.



(a) Configuration (b) mesh

Figure 5. Configuration and mesh of the decomposition comparation test 2.

Our model can be simplified as $\sigma_y = g(d)\left[\lambda(\varepsilon_x + \varepsilon_y) + 2\mu\varepsilon_y\right]$ in this case, which means it is categorized as **tension**.

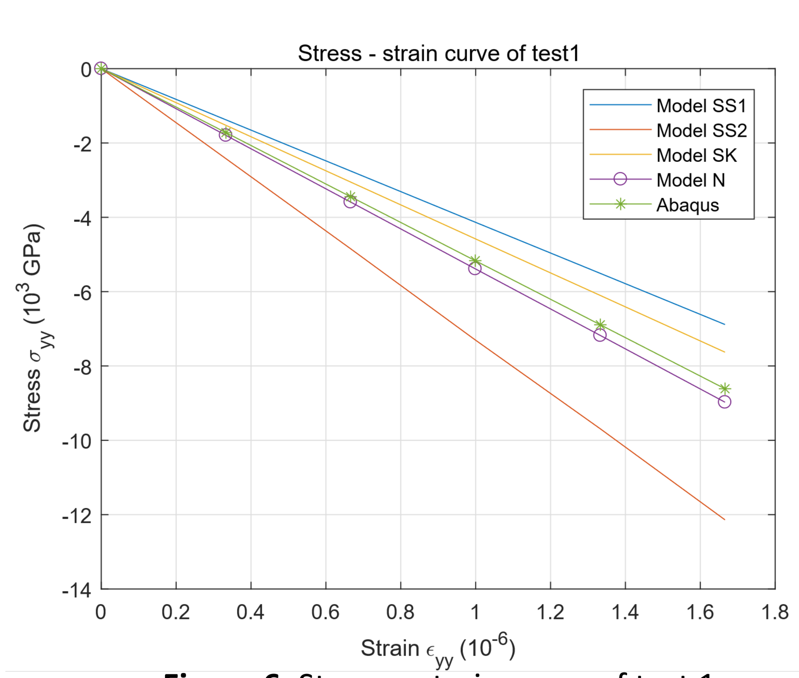


Figure 6. Stress – strain curve of test 1.

Model SS1 and SS2 were proposed by Strobl and Seelig [8], Model SK was proposed by Steinke and Kaliske [6]. Model N is our model and Abaqus result is the reference result. From Figure 6 and 7 we can find that the result of Model N is the closest to Abaqus result, it can be concluded the tension-compression discriminant works well.

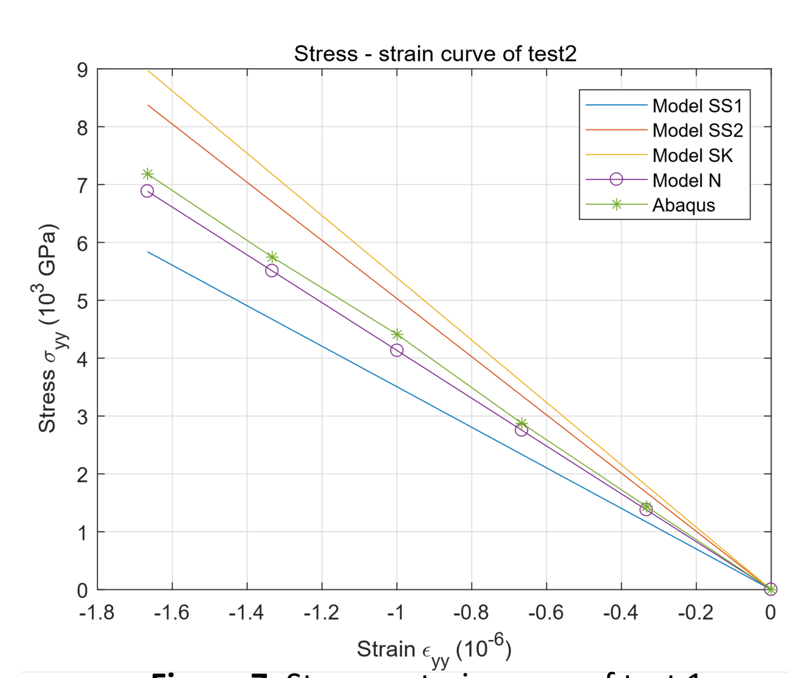


Figure 7. Stress – strain curve of test 1.

Conclusions

We have proposed a homogenization-based phase field model whose macroscopic constitutive relationship is fully determined by the microstructure. The invariant form of our model shows the feasibility of the three-dimensional generalization. According to the result of the numerical examples illustrated in our work, the proposed homogenization-based phase field model shows good performance when compared with existing models, and close results when compared with the benchmark. Additionally, with a tension-compression discriminant, our model handle the mixed load case successfully and give a closer result to Abaqus reference result compared with other models.

Future Directions

In the future, we will establish a model with more complicated microstructures. And we may improve the efficiency of local phase field evolution.

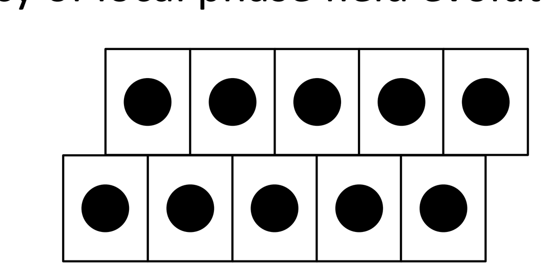


Figure 8. Fiber-reinforced composites

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